Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

*GX*_____

Semester II Examinations, 2003/2004

Exam Code(s)	3BP121
Exam(s)	Third Year Electronic and Computer Engineering
Module Code(s)	EE308
Module(s)	Signals and Communications
Paper No.	1
Repeat Paper	Special Paper Supplemental
External Examiner(s)	Professor S. McLaughlin
Internal Examiner(s)	Professor D.J. Wilcox Dr. J. Breslin

Instructions: Answer 3 questions.
All questions carry equal marks.

Duration	2hrs	
No. of Answer books	1	
Requirements:		
Handout		
MCQ		
Statistical Tables		
Graph Paper		
Log Graph Paper		
Other Material	Yes	Standard Mathematics Tables
No. of Pages	3	
Department(s)	Electronic Engineering	

1.

(a)

(i) What is an LTI system? **[4 marks]**

(ii) Explain the term "orthogonal basis function" for signals. **[4 marks]**

(iii) State Parseval's theorem for periodic functions. **[2 marks]**

(b) Obtain the exponential Fourier series representation of the periodic signal $f(t) = e^{-t}$ over the interval (0, 0.5), which repeats with a frequency of 2 Hz as shown in Fig. 1. Sketch the magnitude and phase spectra of the signal. **[10 marks]**

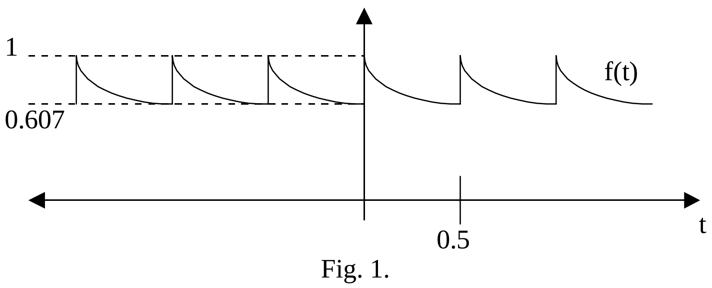

Fig. 1.

2.

(a) Obtain the trigonometric Fourier series representation of the signal $g(t) = t^2$ over the interval (0, 2), which repeats with a frequency of 0.5 Hz as shown in Fig. 2. What is Gibbs' phenomenon, and explain whether it applies to the signal g(t) or not. **[10 marks]**

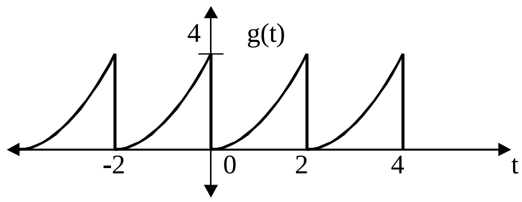

Fig. 2.

(b) Prove that the trigonometric Fourier series of the even function s(t) as shown in Fig. 3 is given by:

$$s(t) = \frac{4}{\pi} \sum_{\substack{n=1 \\ n\,\text{odd}}}^{\infty} \frac{1}{n^2} \cos(nt) \text{ over the interval } (-\pi, \pi).$$

Define half-wave symmetry, and explain if the signal s(t) displays this property or not. **[10 marks]**

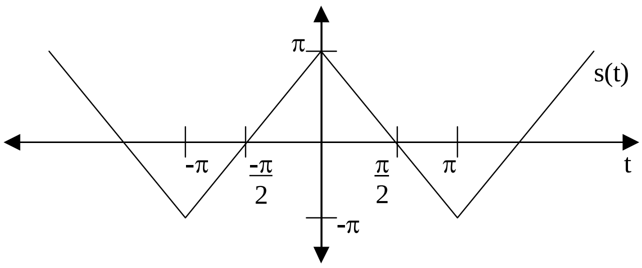

Fig. 3.

[cont'd]

3.

(a) Name and prove the following properties of the Fourier transform:

(i) $\Im\{f^*(t)\} = F^*(-\omega)$. **[3 marks]**

(ii) If $\Im\{g(t)\} = G(\omega)$, then $\Im\{G(t)\} = 2\pi g(-\omega)$. **[4 marks]**

(iii) $\Im\{s(\alpha t)\} = \frac{1}{|\alpha|} S\left(\frac{\omega}{\alpha}\right)$. **[3 marks]**

(b)

(i) Find the Fourier transform of the function $x(t) = (1 - 2e^{-t})[u(t) - u(t-2)]$. **[7 marks]**

(ii) If $\text{rect}(t) \Leftrightarrow \text{Sa}(\omega/2)$, determine the Fourier transform of $\text{Sa}(t/2)$. **[3 marks]**

4.

(a)

(i) Draw the magnitude and phase spectra corresponding to the frequency response of an ideal low pass filter with cutoff frequency ω_c. **[3 marks]**

(ii) What is the principle of causality? Does it hold for the impulse response of an ideal LPF? **[4 marks]**

(iii) Explain the difference between a band pass filter and the passband of a filter. **[3 marks]**

(b) Explain what is meant by the "-3 dB bandwidth" of a filter. If the magnitude of the frequency transfer function of an nth order Butterworth filter is:

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^{2n}}},$$

what is the DC gain? Compute the ratio of the –60 dB to –6 dB bandwidths for the first, second, third and fourth order Butterworth filters with ω_0 = 1 rad/s. **[10 marks]**

5.

(a)

(i) Describe the following physical sources of noise: thermal noise and shot noise. In both cases, give an approximate expression for the power spectral density. **[6 marks]**

(ii) Sketch the PSD and autocorrelation functions for both ideal and bandwidth-limited white noise. **[4 marks]**

(b) By modelling a noisy resistor as a voltage source in series with an ideal resistor, derive an expression for the RMS noise voltage out of a noisy resistor R when it is connected in parallel with a capacitor C. Make use of the fact that the power spectral density of the output noise is an even function of frequency, and also note that:

$$\int_0^\infty \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\tfrac{x}{a}\right).$$ **[10 marks]**